DSC 152 Practice Midterm 2

Solutions and explanations

These solutions correspond to the practice midterm on Lectures 6–11. Equivalent correctly reasoned answers should receive credit. R code may vary while performing the requested procedure.

Question 1

a) $H_0 : \mu_G = \mu_S$ and $H_A : \mu_G \neq \mu_S$, where μ_G and μ_S are the population mean checkout times under Guided and Standard, respectively.

b) `t.test(checkout_sec ~ layout, data = trial)$p.value`

c) The interpretation is incorrect. Assuming the population means are equal and the test conditions hold, the probability of obtaining a t-statistic at least as extreme as the observed one in either direction is 0.080.

d) Observations should be independent within and across groups for the usual standard error to apply. With 12 observations per group, approximately normal population distributions support the t reference distribution. The default Welch test allows unequal population variances.

Question 2

a) The fitted time is $60 - 2(4) = 52$ seconds. The residual is $55 - 52 = 3$ seconds.

b) $t = (-2 - 0)/1 = -2$ with $24 - 2 = 22$ degrees of freedom because the model estimates an intercept and a slope.

c) Fail to reject H_0 because $0.058 > 0.05$. If the population slope is zero, the probability of observing a t-statistic with magnitude at least 2 is about 0.058 under the test conditions.

Question 3

a) B. The estimated two-sided p-value is $(38 + 32)/1000 = 0.070$. Fail to reject H_0 at $\alpha = 0.05$.

b) (a) `FALSE`, (b) `g`, (c) `abs(null_stats) >= abs(obs)`.

c) It estimates the probability, over random reassignments of the layout labels under the null, of obtaining a mean difference with magnitude at least 6 seconds.

Question 4

a) At the same number of prior uses, the fitted mean checkout time is 3 seconds longer with Guided than with Standard.

b) Guided users average three more prior uses, and each additional use is associated with a fitted time 2 seconds shorter. This contributes $-2(3) = -6$ seconds to the unadjusted difference, giving $3 - 6 = -3$ seconds.

c) Substituting $x = x_c + 4$ gives $60 + 3G - 2(x_c + 4) = 52 + 3G - 2x_c$. The intercept of 52 seconds is the fitted mean for Standard users with four prior uses.

d) No. Volunteers chose their layouts, so other differences between the groups may explain the adjusted association.

Question 5

a) The full and reduced models estimate four and two coefficients, respectively. Thus $df_1 = 4-2 = 2$, $df_2 = 72 - 4 = 68$, and $F = \frac{(3112-2720)/2}{2720/68} = 4.9$.

b) Under H_0, the three population mean checkout times are equal at the same number of prior uses. Reject H_0 because $4.9 > 3.13$. There is evidence that at least two adjusted layout means differ.

Question 6

a) (a) `beta1`, (b) `length(x)`, (c) `sd`, (d) `pval < alpha`, (e) `count / reps`.

b) With the 24 prior-use counts held fixed, a true slope of -1 second per prior use, and independent normal errors with standard deviation 6 seconds, about 43% of repeated studies would reject $H_0 : \beta_1 = 0$ at $\alpha = 0.05$.

Question 7

a) Fail to reject H_0 because $0.42 > 0.05$.

b) The increasing residual spread calls the equal variance condition into question. The Shapiro–Wilk result addresses normality, while unequal error variances can make the usual coefficient standard errors and p-values unreliable.

Question 8

a) B. The Guided-minus-Standard difference is $50 - 52 = -2$ seconds with the prompt off and $41 - 49 = -8$ seconds with it on. The interaction estimate is $-8 - (-2) = -6$ seconds.

b) B. The test of $H_0 : \beta_G = 0$ concerns the Guided effect with the prompt off. The prompt and interaction coefficients are unrestricted.

c) $H_0 : \beta_G = \beta_{GP} = 0$. The Guided effect is β_G with the prompt off and $\beta_G + \beta_{GP}$ with it on, and both effects are zero under this null. Fit `reduced <- lm(checkout_sec ~ prompt, data = fact)` and compare using `anova(reduced, full)`. Reject H_0 because $0.0037 < 0.05$. There is evidence that layout affects mean checkout time at one or both prompt settings.