DSC 152 Practice Midterm

Solutions and explanations

These solutions correspond to the practice midterm on Lectures 1–5. Equivalent correctly reasoned answers should receive credit. R code may vary while performing the requested procedure.

Question 1

a) $H_0 : \mu = 18$ minutes; $H_A : \mu \neq 18$ minutes, where μ is the population mean trip duration.

b) `t.test(scoot$duration_min, mu = 18)$p.value`

c) The interpretation is incorrect. A p-value is calculated assuming H_0 is true; it is the probability of observing a test statistic at least as extreme as the one observed. It is not the probability that H_0 is true or that an error occurred.

d) The observations should be independent (and the data should represent a random sample from the target population). For a small sample, the population distribution should be normal so the t-statistic follows its reference t-distribution under H_0; otherwise the nominal Type I Error rate may not be attained.

Question 2

a) Statistical power.

b) The test must reject H_0 on every repeated sample generated under the stated alternative; equivalently, the p-value must fall below α every time.

c) A power of about 0.9999 means that if trip durations truly follow $N(30, 4^2)$ and we repeatedly take samples of size 10 and test $H_0 : \mu = 18$ at level 0.05, about 99.99% of those tests will reject H_0.

Question 3

a) B. The actual Type I Error rate may differ from 0.05 when the distributional condition is not met; it can be inflated or deflated.

b) (a) `18`; (b) `p.value`; (c) `0.05`.

c) It estimates the probability that this t-test rejects H_0 when the null is true for the specified right-skewed population and sample size.

Question 4

a) The signs in order are $-, +, -, +, 0, +, -, +$. The observation equal to 18 is a tie and is omitted from the sign count.

b) There are 4 positive signs, 3 negative signs, and 7 nonzero signs.

c) $2P\{X \leq \min(4, 3)\} = 2P\{X \leq 3\}$ for $X \sim \text{Binomial}(7, 0.5)$, or `2 * pbinom(3, size = 7, prob = 0.5)`. This evaluates to 1.

d) The sign test is about whether values are equally likely to fall above and below the reference value, which corresponds to a population median of 18 under the usual median interpretation. The one-sample t-test tests the population mean $\mu = 18$.

Question 5

a) $2(0.5)^6 = 2/64 = 0.03125$, attained when all six signs agree.

b) The actual Type I Error rate is $2/64 = 0.03125$. At the strict cutoff $p < 0.05$, only the all-positive and all-negative outcomes reject; each has probability $1/64$ under the null. Outcomes with one sign opposite have p-value $2P\{X \leq 1\} = 14/64 = 0.21875$ and do not reject.

Question 6

a) (a) `n`; (b) `18 + delta`; (c) `sd`; (d) `pval < alpha`; (e) `count / reps`.

b) It estimates that about 35% of repeated samples of 12 trips from a normal population with mean 21 minutes and standard deviation 6 minutes would lead the two-sided t-test to reject $H_0 : \mu = 18$ at $\alpha = 0.05$.

Question 7

a) Reject H_0, because $0.003 < 0.05$.

b) This illustrates that statistical significance and practical importance are different. With a very large sample, even a small difference (here, 0.12 minutes) can produce a small p-value. The manager should consider the size and operational meaning of the estimated effect, not the p-value alone.

Question 8

a) B. Fail to reject H_0, because 18 is inside the 95% confidence interval. This follows from the stated CI-inversion rule.

b) B. The bootstrap is nonparametric in how it resamples the observed data, but its confidence intervals are not guaranteed to have exact nominal coverage or Type I Error for every distribution, particularly with small samples.

c) Repeatedly draw a sample of the chosen size from the specified population whose mean is 18. For each sample, construct the same bootstrap confidence interval and record whether it excludes 18. The fraction of intervals that exclude 18 estimates the actual Type I Error rate.