**Quiz 2 - DSC 152, Spring 2026**

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| Full Name: | SOLUTIONS |
| PID: | |

Quiz Time: ◯ 3pm ◯ 4pm

**Instructions:**

- This quiz consists of **4** questions, each worth 20 points (for a total of 80 points). You have **50 minutes** to complete it.
- Please write **clearly** in the provided answer boxes; we will not grade work that appears elsewhere. Completely fill in bubbles and square boxes; if we cannot tell which option(s) you selected, you may lose points.

  ◯ A bubble means that you should only **select one choice**.

  ☐ A square box means you should **select all that apply**.
- Show all work and R code where requested. Partial credit may be awarded.
- You may use one double-sided handwritten sheet of notes. No calculators, no computers.
- Assume we have already run all necessary `library()` calls in R.

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By signing below, you are agreeing that you will behave honestly and fairly during and after this quiz.

Signature:

# Version A

Please do not open your quiz until instructed to do so.

A public health organization is interested in understanding factors that influence recovery outcomes for patients undergoing treatment for tuberculosis (TB), the world's deadliest infectious disease. The organization collects observational data from a sample of patients with active TB enrolled in treatment programs across several clinics. Each row in the dataset represents a single patient's treatment record.

In addition to standard care, some patients are enrolled in a new adherence support program designed to improve medication compliance. The dataset is intended to evaluate how adherence and other patient characteristics relate to recovery time.

The resulting dataset, called `tb_data`, contains the following columns:

- `id` (double): unique identifier for each patient
- `adherence_rate` (double): percentage of prescribed doses taken during treatment (0–100)
- `time_to_recovery` (double): number of weeks until the patient is declared recovered
- `undernourishment` (character): insufficiency of patient's daily caloric intake for maintaining a normal, active, and healthy life (`"yes"`, `"no"`)
- `HIV_status` (character): HIV status of the patient (`"positive"`, `"negative"`)
- `intervention` (character): type of care received, whether standard treatment (`"standard"`) or standard treatment plus the adherence program (`"program"`)

Assume all categorical variables are already coded as factors in R with the following reference levels: undernourishment = no, HIV_status = negative, intervention = standard.

The first five rows of `tb_data` are shown below.

| id | adherence_rate | time_to_recovery | undernourishment | HIV_status | intervention |
|---|---|---|---|---|---|
| 1 | 80.3 | 28.6 | yes | positive | program |
| 2 | 68.3 | 24.7 | no | negative | standard |
| 3 | 74.7 | 19.5 | yes | negative | program |
| 4 | 84.7 | 17.0 | no | negative | program |
| 5 | 91.7 | 17.0 | no | negative | program |

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**Reminder:** Write your **PID** on the top right of this page.

## Question 1

Researchers wanted to see how adherence rates relate to recovery times. They fit the following simple linear regression in R:

```
model1 <- lm(time_to_recovery ~ adherence_rate, data = tb_data)
summary(model1)
```

The abridged output is as follows:

```
Coefficients:
                Estimate Std. Error t value Pr(>|t|)
(Intercept)      38.1950     2.5255  15.123  < 2e-16 ***
adherence_rate  -0.1892     0.0312  -6.063 6.65e-09 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 4.42 on 198 degrees of freedom
Multiple R-squared:  0.1566,        Adjusted R-squared:  0.1523
F-statistic: 36.76 on 1 and 198 DF,  p-value: 6.646e-09
```

**a)** (3 pts) Write a sentence giving the correct interpretation of the slope coefficient in context.

> **Solution:** For every 1 percentage point increase in adherence rate, the expected time to recovery decreases by about 0.1892 weeks, on average.

**b)** (2 pts) What does the intercept represent in this model?

- ◯ The expected recovery time for a patient with 100% adherence
- ● The expected recovery time for a patient with 0% adherence
- ◯ The average recovery time in the dataset
- ◯ The change in recovery time per 1% increase in adherence

**c)** (5 pts) The Residual Sum of Squares (RSS) is defined as:

$$RSS = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

Which of the following are true? **Select all that apply**

■ The RSS is used to find the least squares regression line, by finding the coefficient values that minimize it

☐ This quantity will always be larger when the relationship between x and y is non-linear compared to when it is linear

☐ This quantity only works for simple linear regression, and not multiple linear regression

■ In simple linear regression, the definition of $\hat{y}_i$ is the predicted value of y at the value of $x_i$

■ If we add `HIV_status` to `model1` as a covariate and run a linear model, the RSS from this new model must be less than or equal to the RSS from the original `model1`

**d)** (6 pts) Write R code to produce a residual plot most suitable for checking the **linearity** condition as discussed in class. In doing so, you may extract required quantities from the `model1` object, or write R code to calculate them manually. Do not worry about adjusting axes labels, a title, or any other such elements for this question.

**Solution:**

```
tb_data$resid <- model1$residuals
tb_data$fitted <- model1$fitted.values
ggplot(data = tb_data, aes(x = fitted, y = resid)) + geom_point()
```

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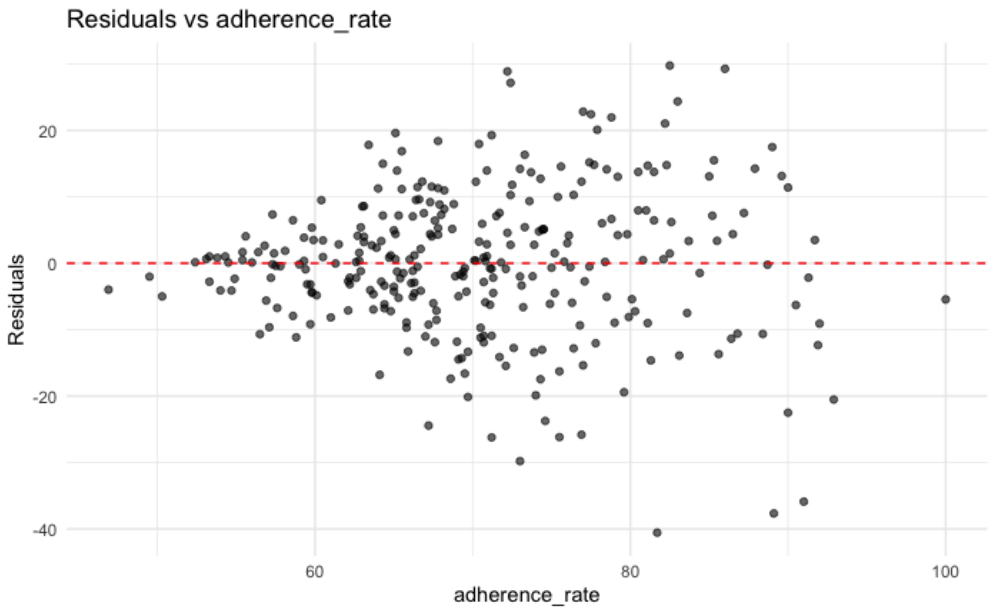
**e)** (4 pts) Researchers wanted to check the validity of using this linear model for statistical inference and create the following diagnostic plot. Are there any conditions that appear to be violated? **Select all that apply**




- ☐ Linearity
- ☐ Independence of observations
- ☐ Normality of $\epsilon_i$
- ■ Equal variance
- ☐ None of the above

## Question 2

Researchers are studying tuberculosis treatment outcomes and are interested in whether adherence to prescribed medication is associated with faster recovery. They also suspect that **undernourishment** may influence recovery time, so they adjust for it in the analysis.

Researchers fit the following model:

```
mod_adj    <- lm(time_to_recovery ~ adherence_rate + undernourishment,
                 data = tb_data)
```

Assume `undernourishment = "no"` is the reference category. Their primary question of interest, then, is whether adherence to prescribed medication is associated with faster recovery, while adjusting for undernourishment.

**a)** (4 pts) What must be true about `undernourishment` for it to be a confounder in the relationship between adherence rate and time to recovery?

> **Solution:** `undernourishment` must be associated with both:
>
> 1. `adherence_rate` (the primary predictor), and
> 2. `time_to_recovery` (the outcome).
>
> That is, undernourishment must relate both to medication adherence behavior and to biological recovery time.

**b)** (3 pts) Suppose the estimated coefficient for `adherence_rate` in `mod_adj` is $\hat{\beta}_1 = -0.08$. Which of the following does this indicate, according to the linear model?

◯ Each 1% increase in adherence causes recovery time to decrease by 0.08 weeks for all patients.

◯ Patients tend to recover 0.08 weeks slower per 1% increase in adherence, among patients with the same undernourishment status.

● Patients tend to recover 0.8 weeks faster per 10% increase in adherence, among patients with the same undernourishment status.

◯ Undernourished patients recover 0.08 weeks faster on average than well-nourished patients.

◯ Adherence has no association with recovery time after adjusting for undernourishment.

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**c)** (5 pts) Suppose the estimated coefficient for `undernourishmentyes` was $\hat{\beta}_2 = 1.5$. Then, the reference category is changed from `"no"` to `"yes"` and `mod_adj` is re-run. Which of the following would be true? **Select all that apply**

■ $\hat{\beta}_2$ would be equal to -1.5

☐ $\hat{\beta}_2$ would be equal to $\frac{1}{1.5}$

☐ $\hat{\beta}_1$ could possibly change and be something different from -0.08

☐ The p-value for $H_0 : \beta_2 = 0$ would be different in this new `mod_adj` vs. the original `mod_adj`

■ The RSS for this new `mod_adj` would be the same as the RSS for the original `mod_adj`

☐ None of the above

**d)** (4 pts) Which of the following are true about the overall Global F-test for the original `mod_adj`?

☐ It tests the $H_0 : \beta_0 = \beta_1 = \beta_2 = 0$

■ It tests the $H_0 : \beta_1 = \beta_2 = 0$

☐ It tests the $H_0 : \beta_1 = \beta_2$

■ It is an inappropriate test to do for the specific question of interest stated at the beginning of this question.

☐ None of the above

**e)** (4 pts) Suppose the p-value for `adherence_rate` in `mod_adj` is 0.003. State the null hypothesis being tested in terms of appropriate parameter(s). Also state the conclusion at $\alpha = 0.05$.

**Solution:** $H_0 : \beta_1 = 0$ (Medication adherence rate has no association with time to recovery after adjusting for undernourishment).

Since $0.003 < 0.05$, we **reject** $H_0$. There is statistically significant evidence that adherence rate is associated with recovery time, after adjusting for undernourishment.

## Question 3

Researchers want to investigate whether adherence rates differed between patients enrolled in the adherence support program and patients that were not. They fit the following model:

```
model2 <- lm(adherence_rate ~ intervention, data = tb_data)
summary(model2)
```

Assume `intervention = "standard"` is the reference category.

**a)** (4 pts) State the null and alternative hypotheses for this question in terms of the appropriate parameter(s) from the linear model, and also in words.

> **Solution:**
> $H_0$: $\beta_1 = 0$. The mean adherence rate is the same for patients in the "standard" and "program" intervention groups.
> $H_A$: $\beta_1 \neq 0$. The mean adherence rates differ between the two intervention groups.

**b)** (5 pts) Write R code to perform a partial $\mathcal{F}$-test to test the hypotheses you wrote in part (a), by first fitting the null model corresponding to the null hypothesis and storing it as `null_model`, and then using `null_model` appropriately. Your output can be the entirety of the partial $\mathcal{F}$-test output; you do not need to isolate the p-value.

> **Solution:**
> ```
> null_model <- lm(adherence_rate ~ 1, data = tb_data)
> anova(null_model, model2)
> ```

**c)** (5 pts) Would an equivalent p-value to that of your partial $\mathcal{F}$-test in (b) appear anywhere in the `summary(model2)` output? If so, describe any and all locations of the equivalent p-value(s) in as much detail as possible. If not, explain why the partial $\mathcal{F}$-test was the only way to get the appropriate p-value here.

> **Solution:** Yes. Because `intervention` has only two levels, the partial $\mathcal{F}$-test is testing the significance of a single coefficient ($\beta_1$). The same p-value would appear in: (1) the row for `interventionprogram` in the coefficients table (as a t-test), and (2) the Global F-statistic p-value at the very bottom of the summary output.

**Reminder:** Write your **PID** on the top right of this page.

**d)** (6 pts) As the above approach in the earlier parts of this question utilized a linear model, it requires the **Equal variance** condition for validity. Write one line of R code to perform a test for the same hypotheses that you wrote in part (a) that does NOT require the **Equal variance** condition. If you cannot think of a way to do it in one line of R code, you may briefly state/describe a statistical test that would work, for partial credit.

**Solution:** `t.test(adherence_rate ~ intervention, data = tb_data)`

## Question 4

Researchers are now interested in whether tuberculosis recovery outcomes differ according to two patient characteristics: **HIV status** ("positive" or "negative") and **undernourishment** ("yes" or "no"). They suspect that the relationship between HIV status and recovery time may differ between patients who are undernourished and those who are not.

The researchers fit the following model:

```
model3 <- lm(time_to_recovery ~ HIV_status + undernourishment +
             HIV_status*undernourishment,
             data = tb_data)
summary(model3)
```

Assume that R uses `"negative"` as the reference level for `HIV_status` and `"no"` as the reference level for `undernourishment`.

**a)** (4 pts) Which of the following best describes why an interaction term is included in this model? **Select all that apply**

☐ To account for baseline differences in recovery time between HIV status groups.

■ To test whether the effect of HIV status on recovery time differs by undernourishment status.

☐ To control for confounding between HIV status and undernourishment.

☐ To test whether both predictors have additive effects on recovery time.

☐ None of the above

**b)** (3 pts) Complete the code below to calculate the mean recovery time for each combination of HIV status and undernourishment.

```
tb_data %>%
  ------------------------------ %>%
  summarize(mean_recovery = mean(time_to_recovery))
```

**Solution:** `group_by(HIV_status, undernourishment)`

**c)** (3 pts) The overall Global $\mathcal{F}$-test p-value reported by `summary(model3)` corresponds to which null hypothesis?

◯ $H_0 : \beta_3 = 0$

● $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$

◯ $H_0 : \beta_0 = \beta_1 = \beta_2 = \beta_3 = 0$

◯ $H_0 : \mu_{\text{yes}} = \mu_{\text{no}}$

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**d)** (5 pts) Suppose researchers are primarily interested in determining whether HIV status has **any** relationship with recovery time, accounting for both the interaction term and undernourishment. If the appropriate p-value is in the `summary(model3)` output, identify as specifically as possible where you would find it. If not, write R code to obtain the appropriate p-value (in this case, any output that contains the appropriate p-value would be acceptable; that is, it does not need to output ONLY the p-value).

**Solution:**

```
null_model <- lm(time_to_recovery ~ undernourishment, data = tb_data)
anova(null_model, model3)
```

**e)** (5 pts) When would it be appropriate to utilize the p-value for the `HIV_statuspositive` coefficient of `model3`? **Select all that apply**

☐ Any time we want to know something about the relationship between HIV status and time to recovery.

☐ When we want to test whether HIV status is associated with time to recovery among those that are undernourished.

■ When we want to test whether HIV status is associated with time to recovery among those that are not undernourished.

☐ When we want to test whether undernourishment status is associated with time to recovery, among those that are HIV positive.

☐ When we want to test whether undernourishment status is associated with time to recovery, among those that are HIV negative.

☐ There is no situation in which it would be correct to utilize this p-value.